On the Localization Problem

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Let us call self-consistency of localization the requirement that the description of a localized state from two different inertial frames should be physically consistent. In previous papers (Kálnay, 1969, 1970) it was shown that this requirement alone gives a precise answer to the localization problem of a quantum relativistic elementary system,† at least in the most important cases (electron, photon, etc.). However, the consequences were heterodox. The self-consistency of localization implied that in relativistic quantum mechanics '... one cannot avoid accepting at least one of the following strong departures from the usual ideas: (i) Position has no sense; (ii) it violates the physical equivalence of inertial frames; (iii) it is the only quantum variable which cannot be represented by an operator; (iv) it is non-Hermitian; (v) some unusual interaction effects do not disappear when the interaction is switched off'. On the other hand, if we accept possibility (iv),‡ then the self-consistency of localization defines an essentially unique position operator whose physical properties are satisfactory (compare Kálnay, 1969, 1970).

The above surprising consequences of the self-consistency of localization in relativistic quantum mechanics makes it reasonable to check it in that part of the quantum theory where the localization problem is completely solved, i.e. in the non-relativistic quantum mechanics of a point particle. To do this is the subject of our present research, whose first results we report here for a spinless free particle.

Using Levy-Leblond's work on the Galilei Group (Levy-Leblond, 1963) we prove that the self-consistency of localization for a non-relativistic particle can be reduced to finding φ_b, g and b_v^3 such that

$$[(\mathbf{p}^3 + mv^3)g(|\mathbf{p} + m\mathbf{v}|) - \mathbf{p}^3 g(|\mathbf{p}|) + i(b_v^3 - b_0^3)]\varphi_b(\mathbf{p} + m\mathbf{v}) = 0$$

where φ_b is a state in momentum representation localized in the point $0 + ib_0^3$ of the third axis, being $b_0^3 \equiv b$ real (we allow, in principle, for imaginary eigenvalues of position in order to see if the self-consistency requirement implies Hermiticity of position in this case); b_v^3 is the transformed of b_0^3 by a Galilei transformation with velocity $\mathbf{v}$; g is a scalar function related to the position operator which can, in principle, be complex.

We can easily prove that $b_v^3 = b_0^3 = 0$, which implies in this case the Hermiticity of the operator: *The self-consistency requirement gives the right answer in the*

† Strictly extended position (cf. Kálnay & Toledo, 1967), is not considered.

‡ This *can* be done in quantum mechanics. Compare Sec. 3 of Kálnay & Toledo (1967). See also the corresponding references quoted in Kálnay (1969).

known case. This gives further confidence to the use of the self-consistency of localization as a way of solving the localization problem in the relativistic case, in spite of the strange consequences that we then obtain.

References

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Relativistic Theory of the Elastic Dielectric

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1. The theories of elastic dielectrics proposed by Toupin (1963) and Grot and Eringen (1966) have been generalized by Salt (1969). The main assumptions and results of the latter work are summarized below.

2. A motion of a body can be represented by a set of three sufficiently smooth functions $X^K = X^K(x^i)$, which map a region of the space-time manifold, called the world tube of the motion, onto the body manifold B (Toupin, 1958). The $x^i, i = 1, 2, 3, 4$, are coordinates of the space-time point x, while the $X^K, K = 1, 2, 3$, are coordinates of the material point X in B. Space-time is a four-dimensional differentiable manifold, endowed with a fundamental tensor g_{ik} that has signature (+,+,+,−). The functions $X^K(x^i)$ representing a motion satisfy the conditions that the matrix $\|\partial_i X^K\|$ be of rank 3, and that the three world vectors $\partial_i X^K$ be space-like.

The world velocity vector w^i is defined at each x in the world tube of the motion by the conditions

$$w_i w^i + 1 = 0$$

$$w^i \partial_i X^K = 0$$

3. At each point of the world tube of a motion an axial scalar mass density ρ may be defined such that mass is conserved:

$$\partial_i(\rho w^i) = 0$$

For any three-dimensional element of extension $d^3 v_i$ in space-time, $\rho w^i d^3 v^i$ is the mass of the element $d^3 X$ of B which is the image of $d^3 v_i$ under the mapping $X^K = X^K(x^i)$.